|Uامتياز فی التحصيليى

# excculencean 

Series


## BOOK PARTS



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## 6 Increasing, Decreasing, Constant functions and Extreme values

1) The function repesented graphically in the opposite figure is increasing in the interval:

(A) $(3, \infty)$
(B) $(1,3)$
(C) $(1, \infty)$
(D) $(-\infty,-2)$

2 In the opposite figure; the function $f(x)$ is decreasing in the interval:

(A) $(0, \infty)$
(B) $(1, \infty)$
(C) $(-\infty,-4)$
(D) $(-4,0)$

3 What is the interval in which the represented function in the opposite figure is increasing?

(A) $(0, \infty)$
(B) $(1, \infty)$
(C) $(-\infty, 1)$
(D) $(-1, \infty)$

(A) 4 at $x=3$
(B) 5 at $x=4$
(C) -1 at $x=1$
(D) 2 at $x=-1$

5 In the opposite figure: the value of $f(a)$ in the interval $[c, d]$ is:

(A) absolute minimum.
(B) local minimum.
(C) local maximum.
(D) absolute maximum.

6 from the opposite figure: the local minimum value of the function $f(x)$ is:

(A) 4
(B) 2
(C) 0
(D) -1
(7) In the opposite figure: the function $f(x)$ in the interval $(2,4)$ is:

(A) Increasing.
(B) Decreasing.
(C) constant.
(D) oscillating.
the function

## Decreasing

If $x_{1}<x_{2}$ then $f\left(x_{1}\right)<f\left(x_{2}\right)$


Example:



## Increasing

If $x_{1}<x_{2}$ then $f\left(x_{1}\right)>f\left(x_{2}\right)$


Attention:
The function can't be described as increasing or decreasing about a point, therefore we use the two brackets (,) when the increasing and decreasing intervals are determined.

## Extreme values


minimum values

## absolute minimum

Existence of local minimum value for the function, if it was the smallest value for the function in its domain.

## local minimum

Existence of a value for the function which is less than all the other values in the interval of its domain.

The points where the function changes its behaviour (increasing or decreasing) forming a top or a bottom, are called critical points.
absolute maximum

| Existence of local maximum |
| :--- |
| value for the function, if it |
| was the greatest value for |
| the function in its domain. |


| Existence of a value for the |
| :--- |
| function which is greater |
| than all the other values in |
| the interval of its domain. |

(1) (A)

The function is increasing in the interval $(3, \infty)$

(4) (D)

The function has a local maximum value at $x=-1$ which equals 2

$f(a)$ is a local maximum value in the interval $[a, d]$
as it represents a top but isn't the highest top, so it won't be absolute maximum value.
..............................................................
(6) C

The function $f(x)$ has a local minimum value which equals 0

The function $f(x)$ is decreasing in the interval $(0, \infty)$

The interval is increasing in the interval $(1, \infty)$

The function $f(x)$ is decreasing in the interval
$(2,4)$.

Cubic function

$$
f(x)=x^{3}
$$



Step function

Quadratic function

$$
f(x)=x^{2}
$$



Absolute value function

## Linear function

$$
f(x)=x
$$



Reciprocal function

## Constant function

$$
f(x)=c
$$

where $c$ is a real number


Square root function
and defined as the largest integer less than or equal $x$.

$$
f(x)=|x|
$$

$$
f(x)=\frac{1}{x}, x \neq 0
$$

$$
f(x)=\sqrt{x}, x \geqslant 0
$$

$$
\mathrm{E} x:[-4]=-4,[-1.5]=-2
$$



## Geometrical Transformations

Horizontal shifting $f(x-h)$

$$
\begin{aligned}
& \text { Vertical shifting } f(x)+k \\
& k<0 \text { downwards } k>0 \text { upwards }
\end{aligned}
$$

## Reflection on the two axis

$$
\text { on } y \text {-axis }
$$

The curve of the function $g(x)=f(-x)$ is a reflection to the curve of the function $f(x)$ about $y$-axis

$$
\text { on } x \text {-axis }
$$

The curve of the function $g(x)=-f(x)$ is a reflection to the curve of the function $f(x)$ about $x$ - axis

$$
g(x)=-\sqrt{x-1}+2 \quad \text { Notice the difference } g(x)=-(\sqrt{x-1}+2)
$$

Shifting the curve of the function $f(x)=\sqrt{x}$ one unit to right followed by reflection on $x$-axis, then make a translation two units upward.

Shifting to the curve of the function $f(x)=\sqrt{x}$ a unit to the right and two units upwards, then reflection on $x$-axis.
(1) B)
(3) (A)

The function $f(x)=[x]$ is defined as the largest integer less than or equal $x$.

$$
\begin{gathered}
\therefore[-2.6]=-3 \\
4 \text { C }
\end{gathered}
$$

$g(x)=|x+3|$
(5) (C)

$$
f(x)=|x+4|-3
$$



The parent function to the function in the graph is $f(x)=x^{2}$


$$
g(x)=-(x-5)^{2}
$$

## 14 Matrices

(1) In the matrix $\left[\begin{array}{ccc}-1 & 3 & 5 \\ 0 & -2 & 4 \\ 6 & 2 & -7\end{array}\right]$, the element $a_{3 \times 2}$ is:
(A) 4
(B) -2
(C) 0
(D) 2
(2) If $A, B$ are two matrices of dimensions $5 \times 3$, then the order of matrix $A-B$ is:
(A) $3 \times 5$
(B) $5 \times 3$
(C) $2 \times 3$
(D) $3 \times 3$
(3) If $A=\left[\begin{array}{cc}-1 & 0 \\ 2 & 3\end{array}\right], B=\left[\begin{array}{cc}0 & -4 \\ 1 & 5\end{array}\right]$, then $3 A-B$ equals:
(A) $\left[\begin{array}{cc}-3 & -4 \\ 7 & 14\end{array}\right]$
(B) $\left[\begin{array}{cc}-3 & 4 \\ 5 & -4\end{array}\right]$
(C) $\left[\begin{array}{cc}-3 & 4 \\ 5 & 4\end{array}\right]$
(D) $\left[\begin{array}{cc}2 & -3 \\ 5 & 4\end{array}\right]$
(4) If the dimensions of matrix $A B$ is $5 \times 8$, and the order of matrix $A$ is $5 \times 6$, then the order of matrix $B$ is:
(A) $6 \times 8$
(B) $8 \times 6$
(C) $6 \times 5$
(D) $5 \times 8$

5 What is the dimensions of the matrix resulted $\left[\begin{array}{lll}a & b & c \\ d & e & f \\ g & h & i \\ j & k & i\end{array}\right] \cdot\left[\begin{array}{l}7 \\ 4 \\ \text { from the following multiplication process: }\end{array}\right]$
(A) $1 \times 4$
(B) $3 \times 3$
(C) $4 \times 1$
(D) $4 \times 3$
(6) The result of multiplication: $\left[\begin{array}{ccc}4 & 0 & -2\end{array}\right] \cdot\left[\begin{array}{cc}2 & -1 \\ -3 & 0 \\ 0 & 4\end{array}\right]$ equals:
(A) $\left[\begin{array}{ll}8 & -12\end{array}\right]$
(B) $\left[\begin{array}{cc}8 & -4 \\ 0 & 0 \\ 0 & -8\end{array}\right]$
(C)
$\left[\begin{array}{c}8 \\ -12\end{array}\right]$
(D) The multiplication
is undefind
(7) The value of $c$, which make the matrix $\left[\begin{array}{cc}-6 & 3 \\ c & 4\end{array}\right]$ doesn't have a multiplicative inverse is:
(A) 8
(B) 6
(C) 2
(D) -8

8 The multiplicative inverse of the matrix $\left[\begin{array}{cc}6 & -3 \\ -1 & 0\end{array}\right]$ is:
(A) $\left[\begin{array}{cc}-2 & 1 \\ \frac{1}{3} & 0\end{array}\right]$
(B) $\left[\begin{array}{cc}0 & -1 \\ \frac{-1}{3} & -2\end{array}\right]$
(C) $\left[\begin{array}{cc}-2 & 3 \\ \frac{1}{3} & 0\end{array}\right]$
(D) $\left[\begin{array}{ll}2 & 3 \\ 1 & 6\end{array}\right]$
(9) If $\left[\begin{array}{cc}14 & -3 \\ 3 t & 0\end{array}\right]=\left[\begin{array}{cc}-7 r & -3 \\ 15 & 0\end{array}\right]$, then the values of $r$ \& $t$ respectively are:
(A) $-2,5$
(B) $2,-5$
(C) $5,-3$
(D) $-7,5$
(10) The result of $2\left[\begin{array}{cc}3 & 5 \\ -6 & 0\end{array}\right]+4\left[\begin{array}{cc}9 & -1 \\ 2 & 3\end{array}\right] \quad$ equals:
(A) $\left[\begin{array}{cc}36 & 9 \\ 4 & 0\end{array}\right]$
(B) $\left[\begin{array}{cc}42 & 6 \\ 4 & 0\end{array}\right]$
(C) $\left[\begin{array}{cc}42 & 6 \\ -4 & 12\end{array}\right]$
(D) $\left[\begin{array}{ll}39 & -6 \\ -4 & 12\end{array}\right]$

## Third-order determinants:

They are determinants of order $3 \times 3$

$$
\left|\begin{array}{lll}
a & b & c \\
d & e & f \\
g & h & i
\end{array}\right|
$$

## The area of the thiangle:

The area of the triangle whose vertices coordinate is $(a, b),(c, d),(e, f)$ is the absolute value of $A$.
as: $\quad A=\frac{1}{2}\left|\begin{array}{lll}a & b & 1 \\ c & d & 1 \\ e & f & 1\end{array}\right|$


## To evaluate the determinant of $3 \times 3$ matrix:

1. Repeat the $1^{\text {st }}$ and $2^{\text {nd }}$ columns to the right.
\(\left.\begin{aligned} \& 2. Find the sum of products of the main diagonal elements and its parallel <br>

\& diagonals, name it s_{1} .\end{aligned} \right\rvert\,\)| $d$ | $e$ | $f$ | $d$ | $e$ |
| :--- | :--- | :--- | :--- | :--- |
| $g$ | $h$ | $i$ | $g$ | $h$ |

3. Find the sum of the products of the secondary diagonal elements and its parallel diagonals, name it $\mathrm{s}_{2}$.
4. Find out $\mathrm{s}_{1}-\mathrm{s}_{2}$ (that is the value of the determinant).

## Cramer's rule:

If $\underline{C}$ is the coefficients matrix of the system $\left\{\begin{array}{l}a x+b y=m \\ f x+g y=n\end{array}\right.$ as $\underline{C}=\left[\begin{array}{ll}a & b \\ f & g\end{array}\right]$ then the solution of the system is: $x=\frac{\left|\begin{array}{ll}m & b \\ n & g\end{array}\right|}{|\underline{C}|}$ and $y=\frac{\left|\begin{array}{ll}a & m \\ f & n\end{array}\right|}{|\underline{C}|}$ that if $|C| \neq 0$

When $|\underline{C}|=0$, then the system has no unique solution.

## (1) (D)

$|$| 4 | 1 | 3 | 4 | 1 |
| :---: | :---: | :---: | :---: | :---: |
| -2 | 3 | 6 | -2 | 3 |
| 0 | 5 | -1 | 0 | 5 |

$=(-12+0-30)-(2+120+0)$
$=-164$

## (2) (C)

(The area of the triangle)
$A=\frac{1}{2}\left|\begin{array}{ccc}0 & 0 & 1 \\ -2 & 8 & 1 \\ 4 & 12 & 1\end{array}\right|$

$=\frac{1}{2} |$| 0 | 0 | 1 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: |
| -2 | 8 | 1 | -2 | 8 |
| 4 | 12 | 1 | 4 | 12 |

$=\frac{1}{2}[(-24)-(32)]$
$=\frac{1}{2}(-56)=-28$
We take the absolute value

$$
|-28|=28
$$



$$
\begin{aligned}
& \left\lvert\, \begin{array}{ccc|cc}
4 & -1 & 1 & 4 & -1 \\
4 & 5 & 3 & 4 & 5 \\
-2 & 0 & 0 & -2 & 0
\end{array}\right. \\
= & (0+6+0)-(-10+0+0) \\
= & 6+10=16
\end{aligned}
$$


$A=\frac{1}{2}\left|\begin{array}{rrr}2 & 4 & 1 \\ -2 & 4 & 1 \\ 0 & -2 & 1\end{array}\right|$

$=\frac{1}{2} |$| 2 | 4 | 1 | 2 | 4 |
| ---: | ---: | ---: | ---: | ---: |
| -2 | 4 | 1 | -2 | 4 |
| 0 | -2 | 1 | 0 | -2 |

$=\frac{1}{2}[(8+0+4)-(0-4-8)]$
$=\frac{1}{2}(12+12)=12$
(5) B
$\left[\begin{array}{rr}-3 & 4 \\ 2 & -5\end{array}\right]$


$$
\because|C|=\left|\begin{array}{rr}
1 & 1 \\
1 & -1
\end{array}\right|=(-1)-(1)=-2
$$

$$
\therefore x=\frac{\left|\begin{array}{rr}
5 & 1 \\
1 & -1
\end{array}\right|}{-2}=\frac{-6}{-2}=3
$$



$$
\because|C|=\left|\begin{array}{rr}
3 & 6 \\
-1 & -2
\end{array}\right|=-6-(-6)=0
$$

$\therefore$ The system has no unique solution.

## ( 8

## Property:

* The determinant whose all elements under or above the main diagonal are zeros, the determinant is called the triangular from.
* The value of the determinant in the triangular form equals the product of the elements of its main diagonal.

$$
\left|\begin{array}{rrr}
3 & -2 & 5 \\
0 & 7 & -6 \\
0 & 0 & 2
\end{array}\right|=2(7)(3)=42
$$

(1) What is the image of the point $J$ resulted from rotation of $\Delta J K L$ with an angle of the measure 270 ?

(A) $(-3,-7)$
(B) $(-7,3)$
(C) $(-7,-3)$
(D) $(7,-3)$

2 The opposite figure shows the quadrilateral $A B C D$ and its image $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ by rotation around the origin point, what is tnemeasure of the angle of rotation.

(A) $90^{\circ}$
(B) $180^{\circ}$
(C) $270^{\circ}$
(D) $360^{\circ}$
(3) What is the image of the point $P(4,5)$ by rotation with an angle $90^{\circ}$ around the origin point?
(A) $(-5,4)$
(B) $(5,4)$
(C) $(-4,-5)$
(D) $(5,-4)$
(4) What is the value of the rotational symmetry of the regular hexagon?
(A) $720^{\circ}$
(B) $180^{\circ}$
(C) $120^{\circ}$
(D) $60^{\circ}$

5 What is the order of the rotational symmetry for the opposite figure?

(A) $135^{\circ}$
(B) $45^{\circ}$
(C) 8
(D) 6
6. If the scale factor of dilation is $k=-3$, then the dilation is:
(A) enlargement
(B) congruent
(C) reduction
(D) translation
(7) If $Q^{\prime} R^{\prime}$ is the image $Q R$ by a dilation of scale factor $K$, and $Q R=6 \mathrm{~cm}, Q^{\prime} R^{\prime}=8 \mathrm{~cm}$, then the scale factor of the dilation $K$ equals:
(A) 2
(B) $\frac{3}{4}$
(C) 1.5
(D) $\frac{4}{3}$

8 Which of the following polygons is not suitable for tiling in the plane?
(A) regular decagon
(B) regular hexagon
(C) square
(D) equilateral tridangle

9 Tiling in the opposite figure, is called:

(A) regular
(B) semi regular
(C) consistent
(D) inconsistent

## Ce Rotation:

It is a geometric transformation, in which each point in the figure rotates by a certain angle and direction around a fixed point called the centre of the rotation.
rotation by angle $270^{\circ} \quad$ rotation by angle $180^{\circ} \quad$ rotation by angle $90^{\circ}$

$$
(x, y) \rightarrow(y,-x)
$$

example:
$(x, y) \rightarrow(-x,-y)$
example:

$(x, y) \rightarrow(-y, x)$
example:


* Rotation by the angle $360^{\circ}$ a round origin returns the figure to its original position.


## el

## The order of the rotational symmetry:

It is the number of times in which the shape and its image are congruent during its rotation from $0^{\circ}$ to $360^{\circ}$

## Example:

The square:
(The order of the rotational symmetry $=4$ ), (The value of symmertry $90^{\circ}$ )

## value of symmetry:

It is the measure of the smallest angle by whic the shape rotate till it becomes congruent to itself.
value of symmetry $=\frac{360^{\circ}}{\text { the order of symmetry }}$

## Dilation:

It is a geometric transfomation enlarges or reduces the shape by a certain reduction.
scale factor of dilation $=\frac{\text { The image length }}{\text { The }}=r$. The value of $r \quad|r|>1 \quad|r|=1 \quad 0<|r|<1$ type of dilation enlargment congruence reduction

## Tiling:

It is a covering of the plane by one shape or a group of shapes.

## consistent <br> If at all vertices the same <br> arrangements for shapes and angles

semiregular
We use two or more than regular polygons
regular

We use one regular polygon

* In tiling, the sum of measures of the total angles around and vertex equals $360^{\circ}$.
(1) (C)

Rotation by the angle $270^{\circ}$
changes the point
$(x, y) \rightarrow(y,-x)$
$\therefore J(3,-7) \rightarrow J^{\prime}(-7,-3)$
(2) B

Rotation by the angle $180^{\circ}$ changes the point
$(x, y) \rightarrow(-x, y)$
$C(8,6) \rightarrow C^{\prime}(-8,-6)$
(3)

Rotation by the angle $90^{\circ}$ changes the point
$(x, y) \rightarrow(-y, x)$
$P(4,5) \rightarrow P^{`}(-5,4)$

> (4) (D)

The order of rotional symmetry for the regular hexagon equals 6 Therefore, value of symmetry

$$
=\frac{360^{\circ}}{6}=60^{\circ}
$$

(5) (C)

The order of the rotational symmetry for the regular octagon equals 8 .
(6) (A)

If $|r|>1$, then the dilation is enlargement.

$$
|-3|=3>1
$$

(7) (D)

$$
\because k=\frac{Q R R^{\prime}}{Q R}=\frac{8}{6}=\frac{4}{3}
$$

8 (A)
Since the measure of the interior angle for decagon equals

$$
\frac{180^{\circ}(10-2)}{10}=\frac{180(8)}{10}=144^{\circ}
$$

and since $144^{\circ}$ isn't a factor of $360^{\circ}$, then the decagon can't be used for tiling the plane.
(9) (C)

This tiling consists of a trapezium and it is an irregular polygon, then the tiling is consistent: as it contains the arrangement of the same shapes and same angles at all vertices.

1. In the opposite figure; find the value of $x$ ?

(A) $30^{\circ}$
(B) $20^{\circ}$
(C) $15^{\circ}$
(D) $10^{\circ}$
(2) What is the circumference of the circle in the opposite figure?

(A) $13 \pi$
(B) $10 \pi$
(C) 13
(D) 7.5
(3) In the opposite figure; the value of $x$ equals:

(A) 9
(B) 6
(C) 4
(D) 3

(A) $232^{\circ}$
(B) $180^{\circ}$
(C) $128^{\circ}$
(D) $64^{\circ}$

(A) $61^{\circ}$
(B) $84^{\circ}$
(C) $119^{\circ}$
(D) $122^{\circ}$

(A) 33 unit.
(B) 36 unit.
(C) 37 unit.
(D) 40 unit.

6 In the opposite figure; A circle is inscribed in triangle RST, what is the perimeter of this triangle.

(A) 22
(B) 12
(C) 4
(D) 3
(8) In opposite figure; the measure of the $\operatorname{arc} \overparen{A B}$ equals:

(A) $60^{\circ}$
(B) $80^{\circ}$
(C) $90^{\circ}$
(D) $120^{\circ}$

## The inscribed angle:

It is an angle whose vertex lies on the circle, and its sides contain two chords of the circle.

The measure of the inscribed angle:
It is half the measure of the subtended arc. $\quad m \angle Q R S=\frac{1}{2} m \overparen{Q S}$


If two inscribed angles in a circle are subtended by the same arc or congruent arcs, then the two angles are congruent.

The measure of the inscribed angle in a semicircle is equal $90^{\circ}$ :


## The tangent:

It is a straight line cuts the circle at one point.

## Angle of tangency:

It is subtended by a tangent and a chord of the circle, its measure equals half the measure of the opposite arc.

(1) B)
$3 x-5=2 x+15$
$x=20$
(2) (A)

$\because m \overparen{J L K}=2\left(116^{\circ}\right)=232^{\circ}$
$\therefore m \overparen{J K}=360^{\circ}-232^{\circ}=128^{\circ}$

$$
\begin{aligned}
& \because m \overparen{Q S}=360^{\circ}-238^{\circ}=122^{\circ} \\
& \therefore m \angle R Q S=\frac{1}{2}\left(122^{\circ}\right)=61^{\circ}
\end{aligned}
$$



The tangent of the circle is perpendicular to the radius at the point of tangency: $\quad \ell \perp A B$


The two tangentsegments drawn to a circle from apoint outside it are congruent:

$$
\overline{A B} \cong \overline{C B}
$$



Since $A B C D$ is cyclic quadrilateral, then the two opposite angles are supplementry.

$$
\begin{aligned}
(23 x+12)^{\circ}+(21 x-8)^{\circ} & =180^{\circ} \\
(44 x+4)^{\circ} & =180^{\circ} \\
44 x & =176 \\
x & =4
\end{aligned}
$$

> (6) B

$$
\begin{array}{r}
\because x+1=5 \\
\therefore x=4
\end{array}
$$

We substitute of $x=4$ in all sides which contains $x$, then the perimeter of the triangle equals 36 unit.

$\because m \overparen{A B}=2\left(60^{\circ}\right)$ $=120^{\circ}$

## 12

## Parabola

(1) The parabola whose equation $x^{2}=8(y-4)$ is opened to:
(A) right
(B) left
(C) down
(D) up
(2) The curve of the parabola and it's axis of symmetry are intersecting at:
(A) focus.
(B) vertex.
(C) directrix.
(D) not intersecting.
(3) The distance between the vertex and the focus of the parabola whose equation $(y-3)^{2}=8(x+4)$ equals:
(A) 2 units
(B) 3 units
(c) 4 units
(D) 8 units
(4) The vertex of the parabola whose equation $(y-5)^{2}=12(x+3)$ is:
(A) $(-5,3)$
(B) $(5,-3)$
(C) $(-3,5)$
(D) $(3,-5)$
(5) Find the length of the latus rectum for the parabola whose equation $(x-1)^{2}=10(y+7)$ ?
(A) 4
(B) 5
(C) 6
(D) 10

6 In the parabola $(y+2)^{2}=-16(x-5)$, the equation of the axis of symmetry is:
(A) $y=-2$
(B) $y=2$
(C) $x=5$
(D) $x=-5$
(7) In the parabola: $(y+5)^{2}=-12(x-2)$, the equation of the directrix is:
(A) $x=-5$
(B) $x=5$
(C) $y=2$
(D) $y=-2$

8 Find the equation of the parabola whose vertex is $(1,-4)$ and its focus is $(3,-4)$ ?
(A) $(x-1)^{2}=-4(y-4)$
(B) $(x-1)^{2}=8(y+4)$
(C) $(y-4)^{2}=-6(x-3)$
(D) $(y+4)^{2}=8(x-1)$
9. Determine the direction of opening of the parabola curve whose equation $y^{2}=-8(x-6)$
(A) Down.
(B) Up.
(C) Left.
(D) Right.

## Parabola:

A parabola is the set of all points $(x, y)$ in a plane that are the same distance from a fixed line, called the directrix, and a fixed point (the focus) not on the directrix.


The

The shape of the curve


$c<0$

The vertex is the midpoint between focus and directrix.

The distance between the focus and the directrix is $=2 c$
(D)
5 (D)

The standard equation of the curve of the parabola whose axis is vertical is

$$
\begin{gathered}
(x-h)^{2}=4 c(y-k) \\
\text { since } c>0
\end{gathered}
$$

Then it opens upwards.


The curve of the parabola and its axis of symmetry are intersecting at the vertex.

The length of the latus rectum $<4 c=10$
(A)
$\because$ The parabola
$(y+2)^{2}=-16(x-5)$
its axis is horizontal.
$\therefore$ The equation of axis of symmetry: $y=k=-2$
7 (B)
(3) (A)

$$
\begin{aligned}
& \because(y-3)^{2}=8(x+4) \\
& \therefore|4 c|=8 \Rightarrow c=2
\end{aligned}
$$



$$
\begin{gathered}
\because(y-5)^{2}=12(x+3) \\
\therefore(h, k)=(-3,5)
\end{gathered}
$$

length
latus rectum

| Axis of | The | Focus |  |  |  |
| :--- | :---: | :--- | :--- | :--- | :--- | | symmetry directrix coord- |  |
| :--- | :--- | :---: | :---: | :---: |
| equation equation inates | direction vertex equation | equation equation inates




The openning of the parabola always directed from the vertex to the focus

from the figure: we get that the axis of the parabola is horizontal, $c>0$ $(y-k)^{2}=4 c(x-h),(y+4)^{2}=8(x-1)$

(9)

The parabola $y^{2}=-8(x-6)$ its axis is horizontal.

$$
\because c<0
$$

$\therefore$ the opening of the parabola is directed to the left.

1) Suppose that $\theta$ is an angle in the standard position such that $\cos \theta>0$, in which quadrant the terminal side of the angle $\theta$ lies?
(A) The first or the second quadrant.
(B) The second or the third quadrant.
(C) The first or the third quadrant.
(D) The first or the fourth quadrant.
(2) What is the exact value of $\sin \theta$ if $\cos \theta=-\frac{3}{5}, 90^{\circ}<\theta<180^{\circ}$ ?
(A) $\frac{-4}{5}$
(B) $\frac{\sqrt{34}}{8}$
(C) $\frac{4}{5}$
(D) $\frac{5}{4}$
(3) If $\angle B$ is an acute angle in the right angled triangle and $\sin B=\frac{5}{13}$, then find the value of $\tan B$ ?
(A) $\frac{5}{12}$
(B) $\frac{12}{13}$
(C) $\frac{5}{6}$
(D) $\frac{25}{12}$
(4) which of the following is equivalent to the expression: $\frac{\cos \theta}{1-\sin ^{2} \theta}$ ?
(A) $\cos \theta$
(B) $\sec \theta$
(C) $\tan \theta$
(D) $\csc \theta$

5 Find the exact value of $\cos 135^{\circ}$ ?
(A) $\sqrt{2}$
(B) $\frac{\sqrt{3}}{2}$
(C) $-\frac{\sqrt{2}}{2}$
(D) $-\sqrt{2}$
(6) What is the exact value of $\sin 240$
(A) $-\frac{\sqrt{3}}{2}$
(B) $-\frac{1}{2}$
(C) $\frac{\sqrt{2}}{3}$
(D) $\frac{\sqrt{3}}{2}$
(7) The reference angle for the angle with measure $150^{\circ}$ equals:
(A) $15^{\circ}$
(B) $30^{\circ}$
(C) $45^{\circ}$
(D) $60^{\circ}$

8 Find the degree measure of the angle with radian measure $\frac{3 \pi}{2}$ ?
(A) $120^{\circ}$
(B) $180^{\circ}$
(C) $245^{\circ}$
(D) $270^{\circ}$

9 The angle $60^{\circ}$ in radian equals:
(A) $\pi$
(B) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$
(D) $\frac{\pi}{6}$

10 Find the length of the arc in a circle of radius 7 cm , if you know that the measure of the angle of its sector is $90^{\circ}$ ?
(A) 11 cm
(B) 12 cm
(C) 13 cm
(D) 14 cm

## Trigonometric functions:

 $\tan \theta=\frac{\text { opposite }}{\text { Adjacent }}=\frac{\sin \theta}{\cos \theta}$ "Tangent $\theta "$

## Reciprocal identities:

$$
\frac{1}{\sin \theta}=\csc \theta \quad \frac{1}{\cos \theta}=\sec \theta \quad \frac{1}{\tan \theta}=\cot \theta
$$

Converting from degree to radian measure:
$\frac{\theta^{\circ}}{180^{\circ}}=\frac{r}{\pi}$ Where $\theta^{\circ}$ is the degree measure,

## Refernce angles:

First quadrant (1) second quadrant (2) Third quadrant (3) Fourth quadrant (4)


$\theta^{\prime}=\theta-180^{\circ}$
$\theta^{\prime}=\theta-\pi$

$\theta^{\prime}=360^{\circ}-\theta$
$\theta^{\prime}=2 \pi-\theta$

## The length of the arc:

Where $S$ is The length of the arc, $r$ is the radius, $\theta$ is the angle of the circle sector by radian measure known that $\left(\pi=3.14\right.$ or $\left.\frac{22}{7}\right)$

$$
S=r \theta
$$


(1) (D)

$\because \cos \theta>0$
as shown in the figure, $\cos \theta>0$ in the first or the fourth quadrant
(2) (C)
$\because \sin ^{2} \theta+\cos ^{2} \theta=1$
$\therefore \sin ^{2} \theta+\left(-\frac{3}{5}\right)^{2}=1$
$\sin ^{2} \theta=1-\frac{9}{25}=\frac{16}{25}$
$\therefore \sin \theta= \pm \sqrt{\frac{16}{25}}= \pm \frac{4}{5}$
but $90^{\circ}<\theta<180^{\circ}$
$(\sin \theta$ is positire in the second quadrant)

$$
\therefore \sin \theta=\frac{4}{5}
$$

(3) (A)

From pythagores theorem, the length of the hypotenuse is 13

$\therefore \tan B=\frac{\text { opposite }}{\text { adjacent }}=\frac{5}{12}$
(4) (B)
$\because \sin ^{2} \theta+\cos ^{2} \theta=1$
$\therefore \cos ^{2} \theta=1-\sin ^{2} \theta$

$$
\therefore \frac{\cos ^{2} \theta}{1-\sin ^{2} \theta}=\frac{\cos \theta}{\cos ^{2} \theta}
$$

by dividing both sides by $(\cos \theta)$

$$
=\frac{1}{\cos \theta}=\sec \theta
$$



$$
\because \cos 135^{\circ}=-\cos \left(180^{\circ}-135^{\circ}\right)=-\cos 45^{\circ}
$$

$$
=-\frac{1}{\sqrt{2}}=-\frac{1}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}}=\frac{-\sqrt{2}}{2}
$$

## ( 6


(7)
$\because$ (reference angle) $\theta^{\prime}=180^{\circ}-\theta$
$=180^{\circ}-150^{\circ}$
$=30^{\circ}$
( (D)

$$
\because \frac{\theta^{\circ}}{180^{\circ}}=\frac{r}{\pi}
$$

$$
\therefore \frac{\theta^{\circ}}{180^{\circ}}=\frac{3 \frac{\pi}{2}}{\pi}
$$

$$
\begin{equation*}
\theta^{\circ}=180^{\circ}\left(\frac{3}{2_{1}}\right)=270^{\circ} \tag{9}
\end{equation*}
$$

$\because \frac{\theta^{\circ}}{180^{\circ}}=\frac{r}{\pi}$
$\therefore \frac{60^{\circ 1}}{180^{\circ}}=\frac{r}{\pi}$
$\therefore r=\frac{\pi}{3}$

## 10 (A)

$\because 90^{\circ}=\frac{\pi}{2}$
$\because S=r \theta=\frac{7\left(\frac{22}{71}\right)}{2}=\frac{22}{2}=11 \mathrm{~cm}$

## 1 Limits

(1) Find the value: $\operatorname{Lim}_{x \rightarrow 3} \frac{2 x+4}{x-1}$
(A) $\frac{1}{3}$
(B) $\frac{1}{2}$
(c) 3
(D) 5
(2) What is the value: $\operatorname{Lim}_{x \rightarrow 4} \frac{\sqrt{2 x+1}-\sqrt{7}}{x-3}$ ?
(A) $3+\sqrt{7}$
(B) $3-\sqrt{7}$
(C) $\sqrt{7}-3$
(D) 3
(3) Find the value: $\operatorname{Lim}_{x \rightarrow-1} \frac{4-\sqrt{x^{2}+x+16}}{x^{3}-1}$ ?
(A) 0
(B) -1
(C) -2
(D) -3
(4) Find the value: $\operatorname{Lim}_{x \rightarrow 4} \frac{x^{2}-16}{x-4}$ ?
(A) -4
(B) 6
(C) 8
(D) 16
(5) Find the value: $\operatorname{Lim}_{x \rightarrow 3} \frac{x^{2}-8 x+15}{x-3}$ ?
(A) 5
(B) 0
(C) -1
(D) -2

6 The value of: $\operatorname{Lim}_{x \rightarrow-4} \sqrt{x+3}$ equals:
(A) 2
(B) 1
(C) -1
(D) Does not exist
(7) The value of: $\operatorname{Lim}_{x \rightarrow \infty} \frac{10 x^{3}-12 x}{5+3 x^{2}-2 x^{3}}$ is:
(A) -5
(B) -2
(C) 0
(D) $\infty$
8. Find the value: $\operatorname{Lim}_{x \rightarrow \infty} \frac{2 x^{2}-5 x}{7-3 x^{3}}$ ?
(A) $\frac{2}{-3}$
(B) $\frac{-3}{2}$
(C) 0
(D) $\infty$
(9) $\operatorname{Lim}_{x \rightarrow \infty} \frac{3 x^{4}-2 x+7}{5 x^{2}+9 x}=$
(A) $\infty$
(B) 3
(C) $\frac{1}{3}$
(D) 0
(10) Evaluate the limit: $\operatorname{Lim}_{x \rightarrow-\infty}\left(x^{3}-2 x^{2}+5 x-1\right)$ ?
(A) 1
(B) $\infty$
(C) $-\infty$
(D) Does not exist
(11) Find: $\operatorname{Lim}_{x \rightarrow-\infty}\left(5 x^{4}-3 x\right)$ ?
(A) 5
(B) $\infty$
(C) $-\infty$
(D) Does not exist
$\qquad$

## The area under the curve:

The area of the region bounded by the curve of the function and the $x$-axis in the interval $[a, b]$ is expressed by ${ }_{a} \int^{b} f(x) d x$.

## In definite integral:

The indefinite integral to the function $f$ is given by the formula $\int f(x) d x=F(x)+C$, where $F(x)$ is an original function for $f(x)$ and $C$ is a constant.

$$
\int x^{n} d x=\frac{x^{n+1}}{n+1}, n \neq-1
$$

## The original functions:

The function $f(x)$ is one of the original functions for the function $g(x)$ if $f^{\prime}(x)=g(x)$

## ( 7 (C)

${ }_{0} \int^{4}(x+k) d x=\left[\frac{x^{2}}{2}+k x\right]_{0}^{4}=\left(\frac{16}{2}+4 k\right)-0$
$\therefore 8+4 k=20 \quad 4 k=12(\div 4) \quad k=3$
(8) (B)

$$
{ }_{1}^{n} 4 x^{3} d x=\left[\frac{4 x^{4}}{4_{1}}\right]_{1}^{n}=\left[x^{4}\right]_{1}^{n}=15
$$

$$
\therefore n^{4}-1^{4}=15
$$

$$
n^{4}=15+1=16
$$

$\therefore n^{4}=2^{4}$
$\therefore n=2$
(9) (C)

$$
\begin{align*}
& \int_{2}^{6}\left(\frac{x^{2}}{x^{2}-1}-\frac{1}{x^{2}-1}+\frac{1}{2}\right) d x \\
= & { }_{2} \int^{6}\left(\frac{x^{2}-1}{x^{2}-1}+\frac{1}{2}\right) d x \\
= & { }_{2} \int^{6}\left(1+\frac{1}{2}\right) d x \\
= & { }_{2} \int^{6}\left(\frac{3}{2}\right) d x=\left[\frac{3}{2} x\right]_{2}^{6} \\
= & \left(\frac{3}{2_{1}}(6)\right)-\left(\frac{3}{2_{2}}(2)\right) \\
= & 9-3=6 \tag{A}
\end{align*}
$$

$\int_{1}^{3}\left(4 x^{3}\right) d x=\left[\frac{4 x^{4}}{4_{1}}\right]_{1}^{3}=\left[x^{4}\right]_{1}^{3}=3^{4}-1^{4}=81-1=80$

## 1 Counting Principle, Permutations and Combinations

(1) The menu in a restaurant has 5 types of main course, 4 types of soups and 3 types of sweets. How many different requests can be made if one chooses one main course, one kind of soup, and one sweet ?
(A) 12
(B) 35
(C) 60
(D) infinite number

2 Nayef Can invite two of his friends to have dinner with him, if he has four friends, by how many ways he can choose them?
(A) 4
(B) 6
(C) 8
(D) 9
3. How many ways can a person enter a mosque which has five doors and exit from a different door?
(A) 120
(B) 60
(C) 25
(D) 20
(4) A car dealer shipped four types of cars, three different colors and two categories, in how many ways can a person choose a car of them?
(A) 24
(B) 18
(C) 12
(D) 9

5 How many ways can 4 people sit a round table?
(A) 24
(B) 12
(C) 9
(D) 6

6 The number of ways 6 people can sit a round table provided that someone sitting next to the window equal $\qquad$
(A) 36
(B) 120
(C) 720
(D) 750

7 If $n!=120$, then $(n-1)!=$ $\qquad$
(A) 16
(B) 24
(C) 36
(D) 90

8 The board of directors of a company consists of 10 members. If Faisal, Mohamed and Muhannad are members of the board, what is the probabilily of selecting three as president, vice president and secretary, respectively, knowing that the selection is random?
(A) $\frac{1}{720}$
(B) $\frac{1}{120}$
(C) $\frac{1}{60}$
(D) $\frac{1}{30}$

9 What is the number of sample elements for selecting two cards with replacement, from a set of numbered cards from 1 to 8 ?
(A) 36
(B) 45
(C) 56
(D) 64

10 Khalid has a math test that asked him to answer 10 questions out of 12 questions, by how many ways can he choose the questions?
(A) 50
(B) 66
(C) 70
(D) 100

The probability value under normal distribution curve:


Normal distribution:


Its graphic repesentation is a curve like a bell, and is symmetrical about the vertical straight line which passes through the mean.

Skewed distributions

Negative skewness (skewed to left)

(D)


Throught the bell curve for the normal distribution then:

$$
\begin{gathered}
P(260<x<340)=(34+34) \%=68 \% \\
\frac{68}{10 \emptyset}(10000)=6800
\end{gathered}
$$

(2) B

Since most of the data is concentrated in the left and a few in the right, then the distribution is positively skewed.

$P(x>3)=(13.5+2+0.5) \%=16 \%$

Positive skewness (skewed to right)

(4) (A)

Since most of the data is concentrated in the right and a few in the left,
then the distribution is negatively skewed.


From the bell curve for the normal gistribution then:

$$
f(10<x<16)=(34+34+13.5) \%=81.5 \%
$$



$$
\begin{align*}
P(x>24) & =(13.5+34+34+13.5+2+0.5) \% \\
& =97.5 \% \tag{D}
\end{align*}
$$

Normal distribution.

9 In the opposite figure: If $m(\overparen{A B})=2 m(\overparen{B C})$ and $\overparen{B C} \equiv \overparen{A D}$ then what is the measure of the arc $B C$ ?

(A) $45^{\circ}$
(B) $90^{\circ}$
(C) $60^{\circ}$
(D) $120^{\circ}$

10 The limit $\operatorname{Lim}_{x \rightarrow 4}(4 x-1)$ equals:
(A) 4
(B) 8
(C) 12
(D) 15

11 If the length of the shadow of the mosque lighthouse is 15 m and the height of the mosque is 2.5 m and the length of its shadow is 1.5 m , then what is the height of the light house?
(A) 9
(B) 15
(C) 25
(D) 40

12 If the radius of a circle is 4 units, and the coordinates of its centre is $(-4,0)$, then which of the following points lie on the circle?
(A) $(4,0)$
(B) $(0,4)$
(C) $(4,3)$
(D) $(-4,4)$

5 If the points: $A(-2,3), B(3,5), C(4,1)$ and $D(x, y)$ represent verticies of the parallelogram $A B C D$, then what is the coordinates of the point $D$ ?
(A) $(-3,7)$
(B) $(7,-3)$
(C) $(-1,-1)$
(D) $(-1,3)$
6. If $\log _{x} 32=5$ then what is the value of $x$ ?
(A) 1
(B) 2
(C) 5
(D) 32
(7) What is the measure of the angle between the two vectors $\langle 2,0\rangle,<3,3\rangle$ ?
(A) $45^{\circ}$
(B) $60^{\circ}$
(C) $90^{\circ}$
(D) $180^{\circ}$

8 What is the derivative of the function: $f(x)=3 x^{2}-5 x+12$ ?
(A) $6 x^{2}-5$
(B) $6 x^{2}-5 x$
(C) $6 x^{3}-5$
(D) $6 x-5$
9. If $y$ varies driectly with $x$, and $y=24$ when $x=8$ then what is the value of $x$ when $y=48$ ?
(A) 12
(B) 16
(C) 20
(D) 24

| Q | A |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | (A) | (B) | (c) | (D) |
| 2 | (A) | (B) | (c) | (D) |
| 3 | (A) | (B) | (c) | (D) |
| 4 | (A) | (B) | (c) | (D) |
| 5 | (A) | (B) | (c) | (D) |
| 6 | (A) | (B) | (c) | (D) |
| 7 | (A) | (B) | (c) | (1) |
| 8 | (A) | (B) | (c) | (D) |
| 9 | (A) | (B) | (c) | (D) |
| 10 | (A) | (B) | (c) | (D) |
| 11 | (A) | (B) | (c) | (D) |
| 12 | (A) | (B) | (c) | ( |
| 13 | (A) | (B) | (c) | (D) |
| 14 | (A) | (B) | (c) | (D) |
| 15 | (A) | (B) | (c) | (D) |
| 16 | (A) | (B) | (c) | (D) |
| 17 | (A) | (B) | (c) | (D) |
| 18 | (A) | (B) | (c) | (D) |
| 19 | (A) | (B) | (c) | (D) |
| 20 | (A) | (B) | (c) | (D) |
| 21 | (A) | (B) | (c) | (D) |
| 22 | (A) | (B) | (c) | (D) |
| 23 | (A) | (B) | (c) | (D) |
| 24 | (A) | (B) | (c) | (D) |
| 25 | (A) | (B) | (c) | (D) |

$$
\because S=\frac{a_{1}}{1-r}=\frac{25}{1-\frac{1}{2}}=\frac{25}{\frac{1}{2}}=50
$$

(2)

$$
\frac{1}{2}\left(\frac{x-1}{x-5}\right)\left(\frac{2 x-2}{x-1}\right)=5
$$

$$
\frac{1}{2}\left(\frac{1}{x-5}\right)\left(\frac{2(x-1)}{1}\right)=5
$$

$$
\frac{x-1}{x-5}=\frac{5}{1}
$$

$$
5 x-25=x-1
$$

$$
4 x=24 \quad(\div 4)
$$

$$
x=6
$$

(3) (C)


$$
x^{\circ}=60^{\circ}
$$



$$
\left\lvert\, \begin{array}{rrr|rr}
i & j & k & i & j \\
1 & -2 & 0 & 1 & -2 \\
2 & 0 & -1 & 2 & 0
\end{array}\right.
$$

$$
=(2 i+0+0)-(-j-4 k+0)
$$

$$
=2 i+j+4 k
$$


$\overline{A E}$ represents a height of the triangle.

$$
\begin{gathered}
\frac{x-1}{x+1}=\frac{6}{5} \\
\because 6 x+6=5 x-5 \\
\therefore x=-5-6=-11
\end{gathered}
$$

## (7) (C)



The quotiet is $2 x^{2}-5 x+3$

## ( D)

The equation of the parabola

$$
x^{2}=8(y+3)
$$

opens upwards.

## (9) (A)

$\because f(x)=\frac{3 x-5}{2}$
$\therefore \frac{y}{1}=\frac{3 x-5}{2}$
$3 x-5=2 y$
$3 x=2 y+5$

$$
x=\frac{2 y+5}{3}
$$

$$
\therefore f^{-1}(x)=\frac{2 x+5}{3}
$$

(10) (A)

difference $<x<$ sum

$$
\begin{aligned}
5-5 & <x<5+5 \\
0 & <x<10
\end{aligned}
$$

## (11) (D)

If we let $n=1$ and $m=3$ (as example)
then $(n+m)^{2}=(1+3)^{2}=16 \quad$ (divisible by 4$)$
and $n^{2}+m^{2}=1+9=10$
hence, $\amalg$ is true, $I$ is true but $\Pi$ is not true

## (12) (B)

$$
n!=4!=4 \cdot 3 \cdot 2 \cdot 1=24
$$

## (13) (D)

The image of the point $(-1,3)$ by reflection in the line $y=x$ is: $(3,-1)$

## (14) (A)

$$
E(3,1) \xrightarrow[y+4]{y-3} \not F(0,5) .
$$

